Solutions to graded problems in Homework 5

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4.5.3

(a)

$$
\begin{aligned}
W &= \left\{ \begin{bmatrix} 2c \\ a-b \\ b-3c \\ a+2b \end{bmatrix} : a,b,c \in \mathbb{R} \right\} \\
&= \left\{ \begin{bmatrix} 0 \\ a \\ 0 \\ a \end{bmatrix} + \begin{bmatrix} 0 \\ -b \\ b \\ 2b \end{bmatrix} + \begin{bmatrix} 2c \\ 0 \\ 3c \\ 0 \end{bmatrix} : a,b,c \in \mathbb{R} \right\} \\
&= \left\{ a \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} + b \begin{bmatrix} 0 \\ -1 \\ 1 \\ 2 \end{bmatrix} + c \begin{bmatrix} 2 \\ 0 \\ 3 \\ 0 \end{bmatrix} : a,b,c \in \mathbb{R} \right\} \\
&= Span \left\{ \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 3 \\ 0 \end{bmatrix} \right\}
\end{aligned}
$$

It is easy to check that the three vectors above are linearly independent, hence a basis for W is:

$$
\mathcal{B} = \left\{ \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 3 \\ 0 \end{bmatrix} \right\}
$$

(b) It follows that $\boxed{\dim(W) = 3}$ (the number of vectors in $\mathcal{B}$)

4.6.9

We know that dim$(Nul(A))$ + $Rank(A) = 6$.

However, we are given that dim$(Nul(A)) = 4$, hence: $Rank(A) = 6 - 4 = 2$.

But, $Rank(A) =$ dim$(Col(A))$ (by definition), hence $\boxed{\text{dim}(Col(A)) = 2}$

4.7.1

(a)

$$\mathcal{C} \overset{P}{\leftarrow} \mathcal{B} = \begin{bmatrix} [\mathbf{b_1}]_{\mathcal{C}} & [\mathbf{b_2}]_{\mathcal{C}} \end{bmatrix} = \begin{bmatrix} 6 & 9 \\ -2 & -4 \end{bmatrix}$$

(b)

$$[\mathbf{x}]_{\mathcal{C}} = \mathcal{C} \overset{P}{\leftarrow} \mathcal{B} \, [\mathbf{x}]_{\mathcal{B}} = \begin{bmatrix} 6 & 9 \\ -2 & -4 \end{bmatrix} \begin{bmatrix} -3 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \end{bmatrix}$$